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LETTER TO THE EDITOR

Direct measurement of the thermal conductivity of a two-dimensional electron gas

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Abstract. The temperature decay along a silicon inversion layer, heated at one end by a DC current, has been measured at liquid helium temperatures by using the electron gas as its own thermometer. The decay profile was obtained by solving the heat diffusion equation with energy loss by phonon emission. Using experimental values of the energy loss rate, the measured temperature decay was fitted to the theoretical profile to obtain values for the thermal conductivity of a two-dimensional electron gas. The values found are in agreement with those predicted by the Weidemann–Franz law.

It is well known that conduction in a two-dimensional electron gas (2DEG) at low temperatures (1–10 K) becomes non-Ohmic for electric fields above about 10 V m^{-1} and that this is due to electron heating effects [1]. When a 2DEG is heated by an electric current, the only mechanism that can remove energy from the electron gas is phonon emission, with characteristic energy loss time τ_E. Energy is redistributed amongst the electrons by inelastic electron–electron scattering in a time τ_{ee} and since $\tau_{ee} \ll \tau_E$ an electron temperature T_e can be defined [2], which can be considerably greater than the lattice temperature as a result of the heating. This electron temperature increases until the energy coming in from the electric field is balanced by that that being lost due to phonon emission. For a degenerate system, the energy balance condition may be written [3] as

$$\tau_E \sigma E^2 = \int_{T_l}^{T_e} c_e \, dT \tag{1}$$

where σ is the conductivity, E is the electric field and c_e the heat capacity per unit area of the 2DEG and T_l is the lattice temperature.

In a relatively high-mobility silicon MOSFET at low temperatures and for $k_F l \gg 1$, the electrical conductivity has been observed (see, e.g., [4]) to have the form

$$\sigma = \sigma_B - \alpha T_e \tag{2}$$

where σ_B is the zero-temperature Boltzmann conductivity and α is a parameter only weakly dependent on electron concentration. Previous workers [5] have attributed just such a linear decrease in conductivity with temperature to variations in the impurity

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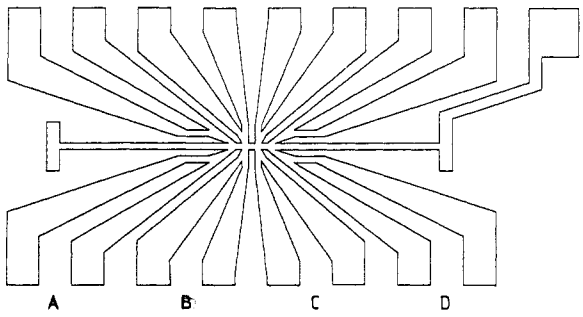

Figure 1. The geometry of the samples used in this work. The central channel is 2 μm wide.

screening. Quantum interference and interaction corrections may also affect the conductivity, the precise temperature for their onset depending on the degree of disorder [6]. In the experiments described in this Letter, however, the important point is that the conductivity varies with electron temperature and so the resistance of the sample itself may be used as a thermometer: the precise nature of the temperature dependence is of secondary importance here. Thus the energy loss rate per unit area ($= \sigma E^2$) may be obtained as a function of electron temperature. A free-electron value for the specific heat may be assumed and equation (1) used to obtain a value [7] for τ_E; alternatively, τ_E may be measured directly and the density of states extracted [8].

The results presented here centre around what happens when only part of the 2DEG is heated (e.g. one end). In this case, rather than one electron temperature being defined for the whole sample, the electron temperature decreases along the sample away from the heating point. The electron gas still emits phonons to maintain energy balance, but the emission rate also decreases as the electron temperature goes down.

The samples used in this work were n-channel (100) MOSFETs and are depicted in figure 1. The main channel is 2 μm wide and 120 μm long; the oxide thickness is 2160 Å and the mobility is 1.2 $m^2 V^{-1} s^{-1}$ at 4.2 K. The junctions of the probes with the channel are 2 μm wide and are separated by 2 μm, giving a separation of 4 μm between the centres of adjacent sets of probes. A large number of probes were used to allow four-terminal measurements of the resistance across the channel at each of the points A, B, C and D and also measurements along the channel. This means that the resistance measured across the channel is only that of a region close to the channel itself, not of the probes, and it can thus be used to obtain the electron temperature at the four points along the channel. Resistance measurements were carried out using a constant AC current and a lock-in amplifier. The electric field was kept below 3 V m^{-1} when calibrating the resistance against temperature, so that the electron gas was in equilibrium with the lattice ($T_e = T_l$). The lattice temperature was assumed to be that of the surroundings, as measured with a resistance thermometer. A low-noise, battery-driven current source was used to provide a DC current for heating the 2DEG when required.

In the first experiments performed, a four-terminal measurement was made of the resistance between probes A and D and calibrated against temperature for two electron concentrations: the corresponding conductances are shown in figure 2. The values of $k_F l$ obtained are 25.1 and 28.7; the system is thus well away from the onset of strong localisation and the approximately linear temperature dependence can be described qualitatively by variations in impurity screening, as described above, with weak localisation leading to a decrease in the conductivity at the lowest temperatures [9]. The sample was then heated by passing a DC current along the channel. The differential

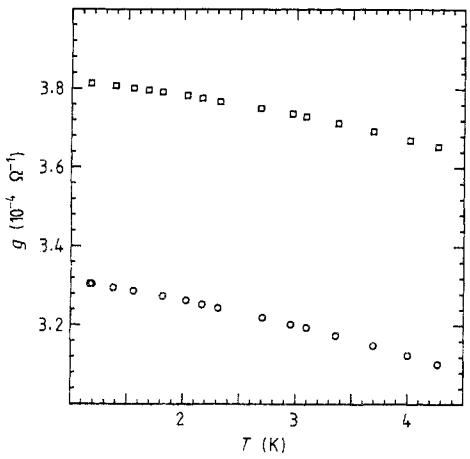

Figure 2. Conductance between probes A and D plotted against temperature for electron concentrations of $0.95 \times 10^{16}\,m^{-2}$ (○) and $1.15 \times 10^{16}\,m^{-2}$ (□). The corresponding values of $k_F l$ are 25.1 and 28.7.

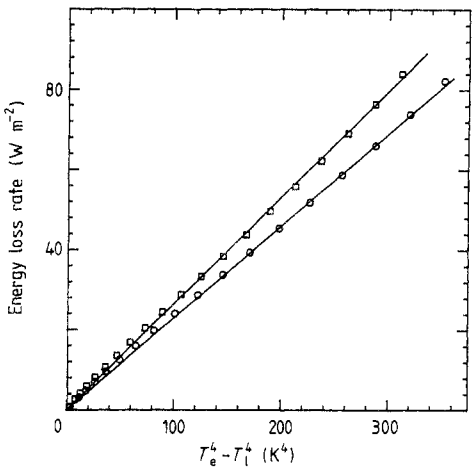

Figure 3. Energy loss rate per unit area from hot electrons plotted against $T_e^4 - T_l^4$ for electron concentrations of $0.95 \times 10^{16}\,m^{-2}$ (○) and $1.15 \times 10^{16}\,m^{-2}$ (□). The full lines are the least-squares linear fits with slopes (0.230 ± 0.002) W $m^{-2}\,K^{-4}$ and (0.266 ± 0.002) W $m^{-2}\,K^{-4}$ respectively.

resistance was then measured as a function of current, averaged over positive and negative directions and numerically integrated to give the resistance, from which the electron temperature was obtained. The energy loss rates calculated are plotted against $T_e^4 - T_l^4$ in figure 3. Such a temperature dependence has been observed and explained [7] by considering deformation potential interaction of the electrons with acoustic phonons in the low-temperature limit for a disordered system. The corresponding time τ_E for an electron temperature of 4 K and lattice temperature of 1 K is of the order of 10^{-9} s, while τ_{ee}, which was derived by fitting the negative magnetoresistance to the theory of Hikami and co-workers [10], is of order 10^{-11} s. This is a sufficient disparity for T_e to be well defined.

In further experiments, the geometry of the sample was used to measure the temperature at the four points along the channel when the electron gas was heated at A by applying a DC current across the channel. Separate calibrations of resistance against electron temperature had to be done for each of the measuring points so that T_e could be accurately obtained.

Figure 4 shows the experimental results for four values of heating current. At A the heating current is constant over a 2 μm width and so the temperature is also assumed to be approximately constant over this region; the temperature obtained from the calibration against resistance is therefore assumed to be that at the edge of this region. For the other measurement points, the temperature obtained is assumed to be that at the middle of the corresponding region.

Heat flows through all branches of the 2DEG in our complex device geometry—probes as well as channel. We therefore analyse the heat conduction in terms of a number of one-dimensional channels, 2 μm wide, in parallel. This allows us to write

$$(d/dx)(\kappa\, dT/dx) = \text{heat loss to phonons} = N_c A(T + T_l)^4 - N_c A T_l^4 \qquad (3)$$

where T is the excess temperature $(= T_e - T_l)$, A is the constant of proportionality

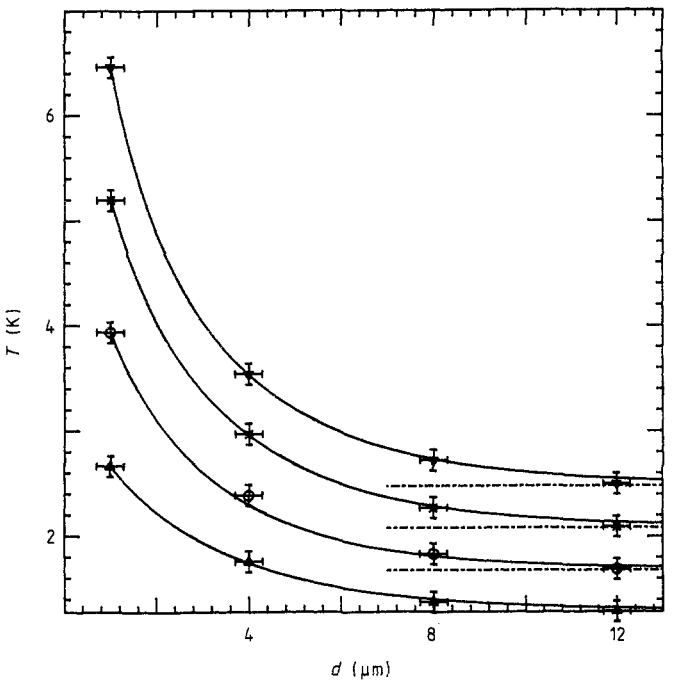

Figure 4. Measured temperature decay profiles for an electron concentration of 0.95×10^{16} m^{-2}. The curves are offset for clarity, with the chain lines representing the lattice temperature of 1.28 K. The full curves show the fit to the theory described in the text, d is the distance (in μm) from the heated region.

derived from the data shown in figure 3, N_c is the number of parallel channels and κ is the thermal conductivity of the 2DEG: this is assumed to have the form

$$\kappa = \gamma T_e = \gamma(T + T_l) \tag{4}$$

where γ is independent of temperature. This follows from a kinetic theory and the linear temperature dependence in the specific heat capacity. We have obtained N_c by measuring the total channel width at distances of 4, 8 and 12 μm from the heating point, yielding a value of 3.8 ± 0.2. We then solved equation (3) analytically to obtain the solution

$$T(x) = T_l[(3\varphi^2 - 2)^{1/2} - 1] \tag{5}$$

where

$$\varphi(x) = \{\alpha[\exp(\eta x) + 1] + \sqrt{3}T_l[\exp(\eta x) - 1]\}/\{\alpha[\exp(\eta x) - 1] + \sqrt{3}T_l[\exp(\eta x) + 1]\} \tag{6}$$

$$\alpha = [(T_o + T_l)^2 + 2T_l^2]^{1/2} \tag{7}$$

$$\eta = (4N_c A/\gamma)^{1/2} T_l \tag{8}$$

and where T_o is the excess temperature at $x = 0$. It is easy to verify that (5) to (8) give the correct behaviour as x tends to zero and infinity. Since T_o must be known before one can use (5) to fit to the data, $x = 0$ must be chosen to correspond to one of the measuring points. In figure 4 T_o is the excess temperature at 1 μm. The validity of the theory may

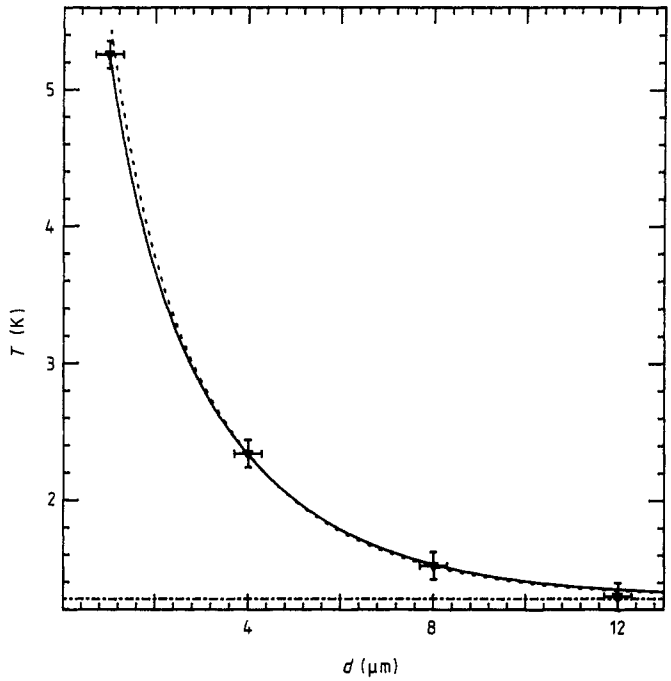

Figure 5. Temperature decay profiles for an electron concentration of $0.95 \times 10^{16}\,m^{-2}$. The full curve shows the theoretical fit using the temperature at 1 μm as the starting point, whereas the broken curve shows the fit using the temperature at 4 μm as the starting point. The chain line is the lattice temperature of 1.28 K. d is as defined in figure 4.

be checked by choosing T_0 to correspond to one of the other measured temperatures and seeing whether the same curve results. An example of this is shown for one value of heating current in figure 5. Virtually the same curve results from taking T_0 at 1 μm and at 4 μm, giving us confidence in our theory.

In obtaining the fits illustrated, $N_c A/\gamma$ was used as a fitting parameter. The value of A is already known from the results shown in figure 3 and N_c has been estimated as described above. Thus γ can be extracted from the experimental temperature decay profiles. Thermal conductivity data are often expressed in terms of the Lorenz number

$$L = \kappa/\sigma T. \tag{9}$$

The theoretical value L_0 is given by the Weidemann–Franz law as

$$L_0 = \pi^2 k_B^2/3e^2. \tag{10}$$

The measured values of L are shown in figure 6 for the data in figure 4, plotted against the temperature at 1 μm. It can be seen that L is very nearly independent of temperature and is close to the Wiedemann–Franz value. This is expected since the Wiedemann–Franz law has been shown to be very generally valid [11]. The value that we extract for the Lorenz number is $(2.3 \pm 0.2) \times 10^{-8}\,W\,\Omega\,K^{-2}$, in good agreement with the theoretical value of $2.45 \times 10^{-8}\,W\,\Omega\,K^{-2}$.

In conclusion, we have developed a technique for investigating the electron–phonon interaction and its effect on the thermal conductivity in a two-dimensional electron gas.

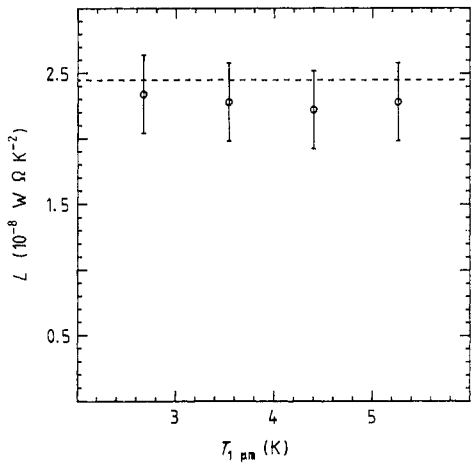

Figure 6. The Lorenz number $\kappa/\sigma T$ obtained from the curves in figure 4 is plotted against the corresponding maximum temperature (the value at 1 μm). The Wiedemann–Franz value is represented by the broken line.

We have measured κ in a silicon inversion layer for hot electrons and found it to be in agreement with the value predicted by the Wiedemann–Franz law. The method could be extended to a wide variety of systems under many different conditions.

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